Biased Campaign Advice

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Abstract

Can biased advisers influence candidates' policy decisions? I develop a cheap talk model where an extremely biased Sender informs candidates about voter preferences. Political competition creates an incentive for some information transmission exclusively to one candidate.This informational asymmetry leads candidates' policy platforms to diverge, but may increase voter welfare. Furthermore, I explore the impact of full commitment for the Sender (information design), and present some cases where the designer optimally gives both candidates identical information. Interestingly, neither symmetry nor precision of information necessarily brings policy closer to the median voter's preference.

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This paper is based on the first and second chapters of my Ph.D dissertation (How Choon (2020)).

1 Introduction

> "I immediately think of interest groups. That's how we gauge our public opinion.... I very rarely am clueless about where that constituency is because of the interest groups keeping me informed..."
>
> — Legislative staffer
>
> Source: Herbst (1998)

A substantial literature has been devoted to understanding political influence — how biased individuals or entities affect policymakers' decisions.[1] Although models are wide-ranging, a large number have focused on strategic information transmission: a biased Sender sends messages to a policymaker (Receiver), who then unilaterally chooses a policy (e.g. Crawford and Sobel (1982), Kamenica and Gentzkow (2011) among many others). However, policymakers are often not social planners but elected politicians. In most democracies, the process of policy creation begins with a "clash of ideas" — a competition between the policy platforms of candidates vying for office. It stands to reason, then, that the effort to influence policy must begin there.

Indeed, from early stages of campaigning, political candidates receive messages from a wide range of informants seeking to influence policy. Some of these exclusively advise one candidate whereas others seek to influence both sides of an election. The former include campaign advisers and partisan interest groups[2] and the latter encompass a number of well-established organizations[3]. There is some

[1]Such entities are sometimes referred to as special interests, and their activities as lobbying. See Grossman and Helpman (2001) for an overview of the literature.

[2]For instance, in the 2016 U.S. Presidential election, Stephen Bannon advised Donald J. Trump but not his opponent, Hillary R. Clinton. In the next election, Democratic presidential candidates benefited from the expertise of progressive advocacy groups while Republican candidates did not (*The New York Times*, "2020 Democrats Import Grass-Roots Activism Into Their Campaign Staffs", March 2019).

[3]In the United States, this may include think-tanks such as the Heritage Foundation and the Brookings Institution, and any news network with a strong editorial stance.

evidence that these special interests provide politicians with information on voter preferences (see Herbst (1998)).

In this paper, I explicitly study how election incentives shape the biased transmission of information about voter preferences. I allow both candidates to receive messages from the same Sender. By presenting a situation where an informant has monopoly over information on voter preferences, I abstract away from any potential competition between information sources. The opposite extreme is examined in the Appendix, with two Senders having diametrically opposed biases and identical information. I show, in that case, that full revelation is an equilibrium. In contrast, the models presented here can be seen as the best-case scenario for a monopoly Sender, or an upper bound on political influence in the absence of competition.

The model is as follows. Two office-seeking candidates play a Downsian election game, where each simultaneously chooses a policy platform, which they commit to implementing if they win.[4] Under majority voting, the policy platform preferred by the median voter must win. However, candidates are uncertain about voter preferences. The Sender possesses this information and is able to send strategic messages to each candidate before they announce their policy platforms. The Sender is extremely biased; she would like the implemented policy to be as high as possible irrespective of voters' true preferences.

Suppose the Sender sends costless messages that have no direct effect on utilities. If she cannot promise not to lie, that is cheap talk. This creates an incentive problem: in the standard cheap talk model, extremely biased Senders always lie, and therefore are never believed (Crawford and Sobel (1982)). In this model, however, political competition makes excessive lying disadvantageous. Suppose the Sender can influence only one candidate. Then, if she falsely convinces that candidate to

[4]As in Downs (1957), I assume that candidates care only about winning and can fully commit to a policy platform.

choose a very high policy, that policy will lose the election anyway, so it will not be implemented. Hence, the Sender had best recommend a policy platform that is just appealing enough to the median voter. I show that an extremely biased Sender is able to convey information, and thus influence policy, in precisely this manner — giving private advice to one candidate only. This prediction mirrors the kind of partnership that we observe in real life between politicians and their advisers.

Contrary to Downs (1957), the winning policy platform is not always equal to (candidates' best guess of) the median voter's most preferred policy. The Sender is sometimes able to bias policy upwards, when "her" candidate is sure to win. Compared to a situation with no information transmission, however, the presence of the Sender may bring the winning policy closer, on average, to the median voter. This result stems from the availability of choice for the voter: when candidates' platforms diverge, an extreme platform can win only if most voters prefer it to the other. Therefore, political competition acts as a moderating force that limits the bias in policy.

To my knowledge, this is the first paper to theoretically establish informational differences between candidates as a source of policy divergence. Most existing models have focused on candidate characteristics (e.g. Besley and Coate (1997) and Kartik and McAfee (2007)) or other features of the election environment, such as heterogeneous districts (Callander (2005)) or convex voter preferences (Kamada and Kojima (2014)), abstracting away from candidates' informational limitations. An important contribution of this paper is in presenting a theoretical link between biased information and political polarization — a link that is intuitive, consistent with anecdotal evidence, and fits a common narrative in modern politics.

The study of cheap talk in elections is not new (e.g. Kartik and Van Weelden (2019)), nor are models with multiple Receivers (Farrell and Gibbons (1989)) or extreme Sender bias (Chakraborty and Harbaugh (2007), Chakraborty and Harbaugh (2010), Lipnowski and Ravid (2017)). However, this paper shows that, in an election context, incentives around the transmission of voter preference information can lead to novel results. Typically, when Senders have state-independent preferences, informative equilibria hinge on an indifference condition — the Sender is exactly indifferent between sending the correct message and lying. Here, with two candidate-Receivers, this is no longer true. When the state[5] is high enough, the Sender strictly prefers recommending a high policy platform to one candidate. This occurs because the state affects the outcome of the election, thus entering indirectly into the Sender's payoff. In this manner, democratic elections sharpen the incentives for information transmission.

The above cheap talk model is most suitable to study Senders who are opportunistic individuals — for instance activists-turned-pundits — who do not necessarily control what information they come across. Such individuals have no explicit commitment to the truth, and may lie if it is in their interest to do so. The model predicts that these individuals will take a partisan role in politics, causing candidates to diverge on the policy spectrum. Nevertheless, their presence does not necessarily harm voters, as the informational benefit may sometimes outweigh the bias they introduce.

However, when knowledge is a result of deliberate research, involving expertly designed surveys and polls, the concept of information design is more suitable. Kamenica (2019) and Bergemann and Morris (2019) provide useful overviews of the literature. In our context, this framework applies mainly to think-tanks and well-established organizations. Commitment power may arise from a combination of disclosure obligations (e.g. Kamenica and Gentzkow (2011)), willful ignorance, and information verifiability (see Gentzkow and Kamenica (2017)). In other words, the Sender decides ex-ante what information to look for, but the information, once

[5]the median voter's preferred policy

obtained, must be shared with candidates — either by obligation or by an incentive-compatible choice. Evidently, this argument applies when the same information is always received by both candidates, i.e. under public information design. In Section 3.2, I show that, in a number of cases, allowing the Sender to design private information (which may differ between candidates) does not affect our predictions.

Consider the case of public information design. After receiving the same message from the Sender, both candidates must arrive at the same posterior belief. They then play the Downs equilibrium, converging to the median of their posterior. Therefore, the Sender's problem can be rewritten as if there were one Receiver who always chose an action equal to the posterior median. I draw a parallel to the gerrymandering literature (Friedman and Holden (2008)), and show that an optimal strategy for the Sender involves revealing the intensity, but not the direction, of voter preferences. In other words, the Sender announces how far voter preferences are from the center of the prior distribution, but not whether they are to the left or to the right.[6] Candidates are indifferent and choose the policy preferred by the Sender.

Compared to the median voter theorem, the results under public information design preserve policy convergence, however candidates always choose policies weakly higher than the median voter's bliss point. The presence of the Sender brings the implemented policy no closer, on average, to the median voter than it would be without information. In other words, the Sender's public messages increase the bias of policy away from the median voter, without improving its precision.

In principle, the Sender may choose to send private messages, or a mixture of public and private messages, under full commitment. Using numerical optimization, I identify a number of cases where public messages are optimal, even when private

[6] A recent working paper by Kolotilin and Wolitzky (2020) examines a class of persuasion problems where the optimal strategy involves pairwise signals (inducing posteriors with at most binary support), a framework which encompasses both Friedman and Holden (2008) and my rewritten problem.

messages are allowed. An example is when the Sender's utility function is linear and candidates have a uniform prior over voter preferences. This is shown by rewriting the Sender's information design problem as a linear programming model, which is solved numerically.[7]

Whilst existing research focuses on information design with voters as the Receivers (Heese and Lauermann (2019), Gitmez and Molavi (2018), Bardhi and Guo (2018)), this paper positions the candidates as the Receivers of information. In doing so, it contributes to the literature on information design in collective decision-making. A technical contribution involves the uncountable state space in this model (the unit interval), where Kamenica and Gentzkow (2011)'s concavification tools do not apply. Although Gentzkow and Kamenica (2016) explore infinite state spaces, they assume that the Receiver's action only depends on his posterior mean — as opposed to the posterior *median* in my model.

The results of this paper contrast the role of a cheap-talk adviser — partisan, polarizing, but potentially welfare-improving — against that of an information designer — reputable, bipartisan, but capable of introducing a much larger bias into policy.[8] The former displays outward signs of biased influence, but the latter type of influence may be hard to distinguish from a truly unbiased policy consensus. Taken together, the two models presented in this paper invite further research into different types of informational lobbying: individuals versus institutions, hearsay versus verifiable research, and their differential effects on polarization and policy.

The rest of the paper is as follows. In Section 2, I set up the cheap-talk model and discuss the main results. Section 3 analyses the analogous information design model. Section 4 discusses the results from a voter welfare perspective, and Section

[7] Many thanks to Stephen Morris for suggesting this method.

[8] The Sender' utility must always be weakly higher under information design than under cheap talk — in most cases strictly higher.

5 concludes.

2 Cheap Talk

To model political competition, I use the Downsian model where two political parties or candidates choose their policy platforms, which are then subjected to the vote of the public. The policy platform closest to the median voter's bliss point wins. I assume that candidates do not know the exact location of the median voter, denoted by θ, but have some common prior over the state space. The Sender, however, knows θ exactly, and is able to send private messages to the candidates before they commit to a policy platform. Candidates are purely office-motivated, whereas the Sender wishes the winning policy to be as high as possible, irrespective of θ.

2.1 Model

There are three players: the Sender, S, and two candidates (Receivers), $i = 1, 2$.

$F : \Theta \to [0, 1]$ is the common prior over the state θ. Assume F has full support on $\Theta = [0, 1]$and continuous density f.

The timeline of the game is as follows:

1. S observes θ. Then S sends a private message m_i to each candidate.
2. Candidate $i \in \{1, 2\}$ observes m_i and chooses $a_i \in [0, 1]$.
3. Election result: If there is no tie, the winner is $w = \arg\min_i |a_i - \theta|$, and the policy $a_w = \arg\min_{a \in \{a_1, a_2\}} |a - \theta|$ is implemented. In case of a tie, $(w,\ a_w)$ is drawn with equal probability from $\{(1,\ a_1), (2,\ a_2)\}$.

Assume that the median voter has symmetric single-peaked preferences with bliss point θ.

Each candidate's payoff is equal to their probability of winning:

$$U_i(a_1, a_2; \theta) = Pr(w = i \,|\, a_1, a_2; \theta)$$

The Sender's utility depends on the winning policy. Voter preferences, represented by θ, do not directly enter the Sender's utility but affect candidates' winning probabilities.

$$\begin{aligned} U_S(a_1, a_2; \theta) &= E(u_S(a_w) \,|\, a_1, a_2; \theta) \\ &= Pr(w = 1 | \, a_1, a_2; \theta) \;\; u_S(a_1) \;\;+\;\; Pr(w = 2 | \, a_1, a_2; \theta) \;\; u_S(a_2) \end{aligned}$$

where $u_S(.)$ is a strictly increasing and continuous. In other words, S would like the winning policy to be as high as possible. (The model is agnostic about the Sender's risk preferences.)

A strategy for S is $\sigma_S : \Theta \to M \times M$, mapping each state to a pair of messages, one to each candidate. Thereafter, I may use $\sigma_{Si}(\theta)$ to denote the Sender's message to candidate i in state θ.

A strategy for each candidate i defines a policy platform for each message: $\sigma_i : M \to [0, 1]$. Therefore, on the equilibrium path, each Receiver chooses a policy equal to $a_i = \sigma_i(\sigma_{Si}(\theta))$.

I consider pure strategy weak Perfect Bayesian Equilibria (henceforth "equilibria") of this game.

2.2 Equilibria

As is usual in cheap talk games, there exists a babbling equilibrium where the Sender sends uninformative messages. The candidates play the Downsian equilibrium, each choosing a policy platform equal to $\theta_{\frac{1}{2}}$, the median of F, irrespective of messages.

Henceforth, I will focus on equilibria where messages convey some information.

In this section, I show that any information transmission by the Sender must treat candidates asymmetrically. The first step is to show that public messages carry no information in equilibrium.

LEMMA 1: Public messages are uninformative. *If we restrict the Sender's strategy to public messages (i.e. $\sigma_{S1} = \sigma_{S2}$ always), then in any equilibrium, both candidates choose $\theta_{\frac{1}{2}}$ regardless of messages.*

If the Sender can send only public messages, both candidates must always update their beliefs in the same manner. In the simple case where every posterior median is unique, both candidates must converge to it. Suppose there were an equilibrium with on-the-equilibrium-path messages inducing distinct policy platforms (posterior medians). Then the Sender would never induce the lower one, hence a contradiction. This intuition is similar to the standard one-Receiver model with large bias (Crawford and Sobel (1982)). The complete proof, which allows for non-unique posterior medians, is provided in Appendix II.

Private messages, however, may convey information. The remaining results in this section provide necessary conditions that must hold true in any equilibrium. Section 2.3 and 2.4 provide examples and establish some sufficient conditions for the existence of informative equilibria.

The following result establishes the asymmetry in candidates' information. Only one candidate may receive informative messages; the other (behaves as if he) learns nothing of θ beyond his prior. This is implied by the following statement.

LEMMA 2. One candidate does not respond to messages. *On the equilibrium path, at least one candidate's strategy must be independent of messages, i.e. $\exists U \in$*

$\{1,2\}$ *such that* $\sigma_U(\sigma_{SU}(\theta)) = a_U \quad \forall\theta$.

Suppose that both candidates' strategies depend on messages — so there must be at least two on-the-equilibrium path policy platforms for each candidate. Consider the lowest of these four policy platforms. The Sender can always do better by inducing each candidate to choose his highest policy. Therefore, she never induces the lowest policy unless it is sure to lose. The candidate can then profitably deviate from this low policy that guarantees a loss.

The next result is easily proven and pins down the uninformed candidate's strategy as the lowest policy on the equilibrium path.

LEMMA 3. Uninformed candidate chooses lowest on-the-equilibrium-path policy. *Let U denote the candidate whose strategy is independent of messages, and let R denote the other. Then* $a_U = \inf A_R$, *where* $A_R = \{a_R : \exists\, \theta \text{ s.t. } a_R = \sigma_R(\sigma_{SR}(\theta))\}$ *is the set of all policies that R may choose after receiving some message.*

The Sender would never induce R to choose a policy strictly below a_U. (She would only do so if the policy was guaranteed to lose, in which case R would not follow the recommendation.) Thus, a_U must lie weakly below R's every on-path policy. Furthermore, a_U must be exactly equal to R's lowest on-path policy, otherwise U could profitably deviate by moving slightly higher (closer to R's policy).

Together, Lemmas 1 and 2 shed light on how information transmission is sustained in equilibrium. The uninformed candidate's policy platform, a_U, is a lower bound on policy. Among the on-the-equilibrium-path policies for the other candidate, the Sender chooses (via the choice of message to R) a policy platform to pit against a_U in the election. A policy platform that is too high above the median voter would lose the election, resulting in a_U being implemented. This incentivizes the Sender to recommend a policy platform that is not too far above the true state θ, so that R wins and the policy is implemented. R follows the Sender's recommendation, as this maximizes his chances of winning.

Whenever an informative equilibrium exists, it gives rise to what Farrell and Gibbons (1989) refer to as "subversion": private messages inform one Receiver but not the other, but public messages inform none. The intuition is completely different, however. In that model, the two Receivers' actions enter the Sender's utility separably. Here, the strategic interaction between Receivers is important. Publicly lying benefits the Sender as both candidates converge to a high policy platform, making voter preferences irrelevant. However, private messages allow for a situation where one candidate is unaffected by the Sender, enabling voters to reject policies too far from θ. This disciplines the Sender to transmit some information truthfully.

It is natural to suspect that these asymmetric equilibria entail some form of polarization. In the absence of the Sender, both candidates are centrists, converging to $\theta_{\frac{1}{2}}$, the median of F. With the Sender, however, one candidate sometimes receives information which leads him to choose higher policies. The other candidate's best response may also diverge from the center. The following lemma states that the uninformed candidate must, in fact, position himself below $\theta_{\frac{1}{2}}$. In other words, the existence of the Sender causes ex-ante identical candidates to diverge from the center, often in opposite directions. An example is provided in Section 2.3.

LEMMA 4. Lowest policy is no higher than the median of F. $a_U \leq \theta_{\frac{1}{2}}$.

One potential benefit to voters is the broader choice available to them. Instead of being presented with identical policy platforms, voters may be given a choice of two distinct policies. In particular, the uninformed candidate's policy provides an alternative against the high policies chosen by the other. This suggests that the winning policy may track the median voter's preferences relatively closely, despite the Sender's extreme bias. This is further discussed in Section 4.

Finally, the extent of information transmission is shown to be limited.

LEMMA 5. Finitely many messages. *Assume $f(\theta) > 0$ for all $\theta \in (0, 1)$. Then, A_R is finite.*

Lemma 5 implies that w.l.o.g. the set of messages sent in equilibrium is finite. This means full revelation is impossible. The proof employs the following intuition. As we move up along the state space, the Sender switches to a higher policy recommendation as soon as that policy is able to win, even by a tiny margin. If the set of meaningful messages (or recommendations) is large, the Sender often induces very close elections where the median voter is almost indifferent between the uninformed candidate's low policy and the other's high policy. But then, the uninformed candidate can deviate slightly upwards and win all these close elections.

2.3 Uniform Prior

To provide a more detailed characterization of equilibria, it is useful to add assumptions about the prior distribution over θ. To begin, I characterize equilibria for the uniform distribution to gain some intuition. The result below states that, on any equilibrium path, no more than two policy platforms are ever chosen.

LEMMA 6. Two messages. *If $\theta \sim Uniform[0, 1]$, then A_R (the set of possible policy platforms on the equilibrium path) is at most a doubleton.*

In equilibrium, the informed candidate chooses one of two policy platforms in response to the Sender's message. W.l.o.g. the information that is transmitted is extremely coarse — taking the form of a two-cell partition of Θ. Lemma 7 characterizes an intuitive class of equilibria, reminiscent of Crawford and Sobel (1982), where the cells are intervals of the state space.

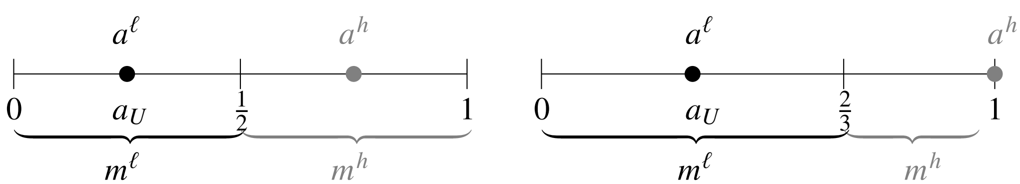
LEMMA 7. Interval equilibria. *If $\theta \sim Uniform[0,1]$, the following is an equilibrium iff $a^\ell \in [\frac{1}{4}, \frac{1}{3}]$ and $a^h = 3a^\ell$.*

Fix some arbitrary $m_U \in M$ and a_ℓ, a_h satisfying the conditions given above. Also fix a partition $\{M^\ell, M^h\}$ of M, and arbitrary elements $m^\ell \in M^\ell$ and $m^h \in M^h$.

$$\sigma_S(\theta) = \begin{cases} (m_U, m^\ell) & \text{if } \theta < \frac{a^\ell + a^h}{2}, \\ (m_U, m^h) & \text{otherwise.} \end{cases}$$

$$\sigma_U(m) = a_U = a^\ell \quad \text{for all } m \in M, \quad \text{with prior belief after every } m.$$

$$\sigma_R(m) = \begin{cases} a^\ell & \text{for all } m \in M^\ell, \quad \text{with belief } \theta \sim Uniform[0, \frac{a^\ell + a^h}{2}], \\ a^h & \text{for all } m \in M^h, \quad \text{with belief } \theta \sim Uniform[\frac{a^\ell + a^h}{2}, 1]. \end{cases}$$



(a) If $a^\ell = \frac{1}{4}, \quad a^h = \frac{1}{2}$.

(b) If $a^\ell = \frac{1}{3}, \quad a^h = 1$.

Figure 1: Equilibria where messages denote intervals of the state space

These equilibria have a simple partitional structure, and can be completely characterized by the two induced policy platforms, a^h and a^ℓ. Lemma 3 immediately implies that the uninformed candidate chooses the lower policy platform, a^ℓ. Furthermore, the cutoff between the two intervals is at the midpoint of a^h and a^ℓ. In other words, the Sender adopts the pragmatic strategy of inducing the higher policy platform if and only if it wins against the lower policy platform. If θ is below that cutoff, a^h would lose, so the Sender might as well induce a^ℓ. The informed candidate, R, always follows the Sender's recommendation, as this maximizes his chances of winning.

However, not all equilibria feature a partition of the state space into intervals. In particular, the equilibrium that gives the Sender the highest utility involves a different informational structure. This is shown in the following lemma.

LEMMA 8. Sender-optimal equilibrium. *If $\theta \sim Uniform[0,1]$, then the following is an equilibrium, and it achieves the highest equilibrium expected payoff for the Sender.*

Fix some arbitrary $m_U \in M$. Also fix a partition $\{M^1, M^2\}$ of M, and arbitrary elements $m^1 \in M^1$ and $m^2 \in M^2$.

$$\sigma_S(\theta) = \begin{cases} (m_U, m^1) & \textit{if } \theta \in [0.25, 0.75), \\ (m_U, m^2) & \textit{otherwise.} \end{cases}$$

$$\sigma_U(m) = 0.5 \quad \textit{for all } m \in M, \quad \textit{with prior belief after every } m.$$

$$\sigma_R(m) = \begin{cases} 0.5 & \textit{for all } m \in M^1, \quad \textit{with belief } \theta \sim Uniform[0.25, 0.75), \\ 1 & \textit{for all } m \in M^2, \quad \textit{with posterior given } \theta \in [0, 0.25) \cup [0.75, 1]. \end{cases}$$

If u_S is linear, this equilibrium gives the Sender expected utility $E_\theta(U_S(\boldsymbol{\sigma}; \theta)) = 0.625$.

The two messages convey information about the *extremeness* of voter preferences: message m^1 is sent if θ is moderate, and m^2 is sent if θ is extreme. After receiving the extreme message, the informed candidate is unsure whether voters

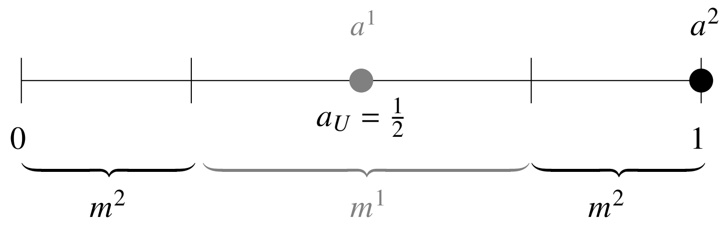


Figure 2: Sender-optimal equilibrium

favor extremely low or extremely high policies, so he might as well choose a very high policy (equal to 1), and win half of the time. The uninformed candidate always chooses the moderate policy ($a_U = \frac{1}{2}$), and so does the informed candidate when he receives the moderate message.

Conveying information about the extremeness — but not the direction — of voter preferences is, in general, a good strategy for the Sender. A similar structure is discussed in Section 3, under public information design, where the Sender reveals to both candidates exactly how far θ is from its median.

The next lemma states that both the Sender and the informed candidate benefit from information transmission.

LEMMA 9. Sender and informed candidate both benefit from cheap talk. *If $\theta \sim Uniform[0, 1]$ and u_S is linear, then any informative equilibrium gives weakly higher utility to the Sender and to candidate R than the uninformative equilibrium.*

This finding is consistent with existing cheap talk models. It also has the intuitive implication that the better-informed candidate wins more often than his opponent.

2.4 Skewed Prior Distributions

In the example above, the prior distribution has no skewness. However, skewness matters for information revelation, because it affects candidates' incentive to deviate. In this section, I consider three classes of prior distribution which illustrate this point.

LEMMA 10. Increasing, decreasing and symmetric prior.

(a) If f is strictly increasing in θ, all equilibria are uninformative.

(b) If f is weakly decreasing in θ, an informative equilibrium exists.

(c) If f is single-peaked and symmetric around $\theta = \frac{1}{2}$, an informative equilibrium exists.[9]

In any informative equilibrium, one candidate, U, must choose a low policy platform under his prior belief about θ. Therefore, higher policy platforms must not be too attractive, or U will profitably deviate by choosing a higher policy platform.

One interpretation is that the Sender influences policy when her information carries unexpected "good news", i.e. when voters favor higher policies than expected under the prior. If prior beliefs are based on past data, then biased advisers are likely to be most influential right after a major shift in voter preferences.

Lemma 10 suggests that candidates' personal advisers will often exhibit eccentric biases which (ostensibly) run counter to voters' inclinations.[10] The adviser will tend to pull the candidate towards a policy platform that, according to the common prior, is unlikely to win — "bad advice", from the perspective of an uninformed observer.

[9]Note that the uniform distribution fits both (b) and (c). In fact, the uniform distribution is at the boundary between increasing and decreasing, so that *all* informative equilibria are doubleton partitions — the coarsest possible information.

[10]The Sender is biased towards the tail of the prior distribution.

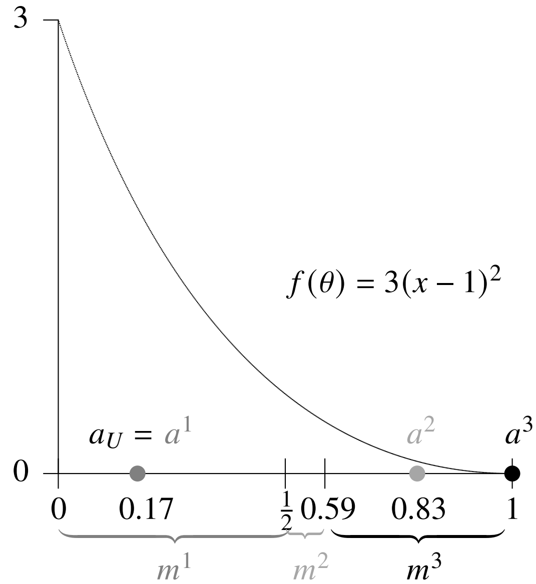


Figure 3: A cheap talk equilibrium with decreasing f

The candidate follows this advice, which may lead to an upset win, leading others to recognize that voter preferences were, in fact, more extreme than expected.

For example, Figure 3 below illustrates a decreasing prior density, with an equilibrium where the Sender sends three distinct messages. Notice that messages m^2 and m^3 induce policy platforms (a^2 and a^3) that are very far into the tail of the prior distribution. These policy platforms do win, however, when $\theta > \frac{1}{2}$. When this occurs, the two candidates are highly polarized, with one candidate defying the conventional wisdom to win, and the winning policy is even more extreme than the median voter's wishes.

3 Information Design

This section studies the problem faced by a Sender who fully commits ex-ante to the information she will send to candidates. Commitment power may arise from strong reputational incentives (not modeled here) or from disclosure obligations, such as in the seminal Kamenica and Gentzkow (2011) example, where a prosecutor decides what evidence to look for but is legally bound to reveal the outcome of any investigation. In another interpretation, the Sender commissions an experiment, the design of which is publicly observed, then decides whether to publish the resulting reports. Gentzkow and Kamenica (2017) show this leads to predictions that are identical to those of a Bayesian persuasion model. In the presence of multiple Receivers, these arguments equivalently hold for an information designer who can only choose public signals.

However, under private information design, the assumption of full commitment is stronger and its defense less straightforward. For instance, the Sender may be able to delegate the collection of information to two independent contractors, who then each privately informs one candidate. In this scenario, the Sender must not be allowed to access any private information after it is collected, lest she is tempted to reveal additional information to one of the candidates ex-post (which would violate the full commitment assumption). In most real-world situations, the assumptions required for private information design are unlikely to be satisfied.

In this section, I set up the general formulation of Sender's problem. I then restrict attention to public messages. Thereafter, I present some cases where this restriction is benign.

The Sender designs a message strategy, or signal structure, that specifies a joint distribution over messages contingent on the state of the world. After receiving a message, each candidate updates their belief about θ and chooses a policy platform. A candidate's strategy must be optimal given their belief about θ and the other candidate's strategy. Assuming the Sender can choose the most favorable equilibrium, we can write this as a maximization problem subject to obedience constraints.

The Sender chooses a message strategy, which specifies a joint distribution

$p(.|\theta) : M \times M \to [0,1]$ for $\theta \in \Theta$, and a strategy profile for candidates, $\{\sigma_i\}_{i=1,2}$, to maximize

$$E(u_S(a_w))$$

where a_w is the winning policy defined as previously, subject to the following obedience constraints.

For each i, for all m_i that are sent with non-zero probability for some θ,

$$\sigma_i(m_i) \in \arg\max_{a_i} \int_{m_j} \int_{\theta} Pr(i \text{ wins } |a_i, \sigma_j(m_j), \theta) \; Pr(m_j, \theta \,|m_i) \; \mathrm{d}\theta \; \mathrm{d}m_j$$

where $Pr(m_j, \theta \,|m_i) = \frac{p(m_i, m_j|\,\theta) \; f(\theta)}{\int_{m_j} \int_{\theta} p(m_i, m_j|\,\theta) \; \mathrm{d}F(\theta) \; \mathrm{d}m_j}$ is i's joint posterior belief over m_j and θ.

Thus, player i updates his belief not only of the state of the world, but also of what information the other candidate might have received. The obedience constraint requires i's policy to be optimal given this updated joint belief.

3.1 Public Messages with Full Commitment

Suppose the Sender designs a public message strategy. Then the following constraint is added:

$$p(m_i, m_j|\theta) = 0 \quad \text{for all} \quad m_i \neq m_j$$

The candidates have a common prior and observe the same signal. Thus they arrive at the same posterior and each must choose a median of this posterior, as in Downs (1957). The best equilibrium for the Sender is where both candidates converge to the highest median for each posterior.[11]

[11]There are different ways of dealing with non-unique posterior medians. One is to only consider the highest (most favorable) median, i.e. $\hat{\theta}_m(s) = \max \theta_{\frac{1}{2}}(s)$, or the lowest (least favorable). Another

The Sender's problem can be rewritten thus:

Choose $p(.|\theta) : M \to [0,1]$ to maximize

$$E_m(u_S(\max \theta_{\frac{1}{2}}(m)))$$

where $\theta_{\frac{1}{2}}(m) \equiv \{t \in \Theta : \ Pr(\theta \leq t|m) \geq \frac{1}{2} \text{ and } Pr(\theta \geq t|m) \geq \frac{1}{2}\}$ is the set of medians of the posterior belief given message m.

First, I will argue that this bears some similarities to a gerrymandering problem, which may be described as follows. A principal decides how to allocate a given population with a given preference distribution $\mathcal{F} : \theta \to [0,1]$ into districts of a given size. Each district then elects a representative, and the principal wishes to maximize the average ideology of the representatives.[12] Under the median voter theorem, the elected representative for each district m has the median ideology for that district, $\theta_{\frac{1}{2}}(m)$.[13]

If we consider the prior distribution $F(\theta)$ as analogous to the voter population $\mathcal{F}(\theta)$ in the gerrymandering problem, and fix some finite support for $p(.|\theta)$, then the Sender's choice of $p(m|\theta)$ is equivalent to allocating voters of a given preference θ to each district $m \in M$. A key constraint in both problems is the requirement that the posteriors (or preferences within districts) must average to the prior (population) distribution. The main difference here is that the support of p, instead of being a fixed number of districts of equal size, can be chosen to be any (potentially uncountable)

is to restrict the signal structure so that only posteriors with unique median are allowed. The solution that will be introduced in Theorem 1 can be easily approximated by a signal structure where all posteriors have a unique median. Therefore, the assumption that candidates will choose the highest median is w.l.o.g.

[12]The simplest form of this is when preferences are binary, i.e. $\theta \in \{0,1\}$, where 0 represents a Democrat and 1 a Republican voter. The principal allocates voters so as to elect as many Republican representatives as possible. The optimal solution famously involves "packing and cracking", whereby some districts are overwhelmingly Democrat and some have just enough Republicans to make a majority.

[13]Note this formulation is slightly different from Friedman and Holden (2008).

set of messages. The Sender's problem therefore has more degrees of freedom than a gerrymandering problem.

This connection with the gerrymandering literature is, however, helpful in formulating the optimal signal structure, which is a limiting case of the solution found by Friedman and Holden (2008).

THEOREM 1. Optimal public information design. *The following signal structure is a solution to the Sender's public information design problem, and achieves the maximal utility equal to* $E(u_S(\theta) \mid \theta > \theta_{\frac{1}{2}})$.

$$m^*(\theta) = \left| F^{-1}(\theta) - \frac{1}{2} \right|$$

$$p(m|\theta) = \begin{cases} 1 & \text{if} \quad m = m^*(\theta), \\ 0 & \text{otherwise} \end{cases}$$

$$\sigma_i(m) = F^{-1}(\frac{1}{2} + m) \quad \text{for all } i \in \{1, 2\},\ m \in M$$

In other words, the Sender's optimal message strategy perfectly reveals θ's quantile distance from the median of F. The Sender optimally reveals exactly how extreme voter preferences are, but not in which direction. Faced with this uncertainty, candidates might as well choose the higher out of the two possibilities.

Figure 4 illustrates the optimal strategy under the uniform prior. There are uncountably many messages on the equilibrium path, here indexed by hue (light to dark on a grayscale). Each message induces a corresponding policy platform (to which both candidates converge), shown by a circle of the same hue.

Note that the optimal strategy is not unique. Also optimal is any other pairwise

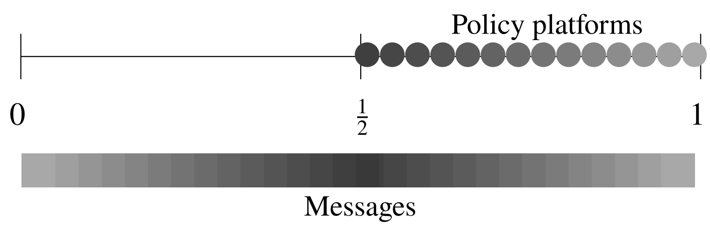


Figure 4: Public information design with $\theta \sim Uniform[0, 1]$.

matching of states, one above and one below $\theta_{\frac{1}{2}}$, where the posterior belief assigns equal mass to each of the two states. Although different pairwise matchings invite different interpretations in terms of real-world messages, the properties discussed below apply to all.

These strategies are weakly stochastically dominant, being optimal for *any* strictly increasing u_S.

The Sender achieves a utility equal to $E(u_S(\theta) \mid \theta > \theta_{\frac{1}{2}})$. Cut the prior distribution in half at the median, then throw away the lower half. Take the expectation over the remaining top half, and that is the Sender's utility. Note that it is always strictly better than babbling. This is in contrast to cheap talk, where information revelation is impossible under some priors, and with the typical persuasion problem, where gains from persuasion are not guaranteed.

Interestingly, the entire lower half of the prior is irrelevant to the Sender. This should not be surprising, given the parallel to gerrymandering, since a gerrymander can always suppress up to half of the electorate by placing them in districts where they are in the minority. Any change in F that preserves the upper tail leaves the Sender's utility unchanged. If F is symmetric, then the more polarized F is, the better.

How does the strategy in Theorem 1 compare to the cheap talk equilibria? Under information design, the Sender has full commitment over messages, which is always better (ex-ante) than cheap talk. However, this may not hold with the additional restriction to public messages. In the uniform-prior case with linear Sender utility, a quick calculation shows public information design to be strictly better than any cheap talk equilibrium, with $U_S = 0.75$ and a maximum of $U_S = 0.625$ respectively. Nevertheless, both compare favorably to the prior median of 0.5 (the Sender's utility under babbling).

The following lemma extends this result to any single-peaked symmetric prior and any risk-averse Sender. This implies that the Sender values full commitment power over the ability to send private messages.

LEMMA 11. Commitment versus private messages. *Suppose that $u_S(.)$ is weakly concave and that f is single-peaked and symmetric. Then the Sender's utility under public information design is strictly higher than under any cheap talk equilibrium.*

3.2 Private Messages with Full Commitment

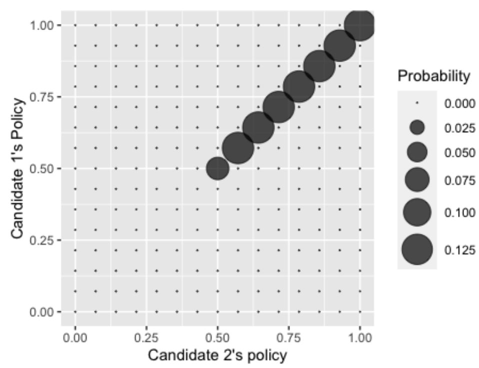
The Sender's problem becomes significantly more complex once private messages are allowed. However, if discretized, it can be solved by linear programming, for a given prior distribution. By an argument similar to the revelation principle, the Sender's problem can be rewritten w.l.o.g. as choosing the candidates' *policies* subject to obedience constraints. In this formulation, public messages are optimal if and only if the policies chosen by the Sender for the two candidates are perfectly correlated.

Solving numerically for the optimal Sender strategy, I find that, in a number of cases, public messages are optimal even when private messages are allowed.[14] For

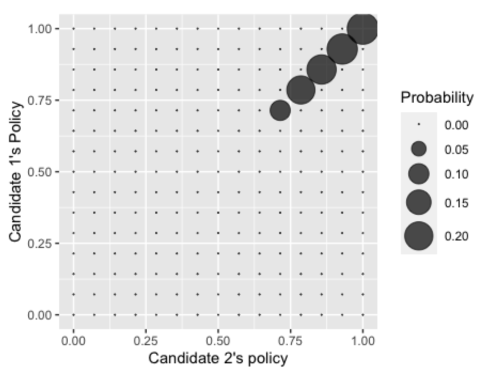
[14] The code in R is available online at https://theahowchoon.com/s/code.pdf

example, public messages are optimal when the Sender's utility is linear and Θ is uniformly distributed over the unit interval. The same is true under a variety of prior distributions, both positively and negatively skewed. This suggests that the assumption of public information design is not as restrictive as one might expect.

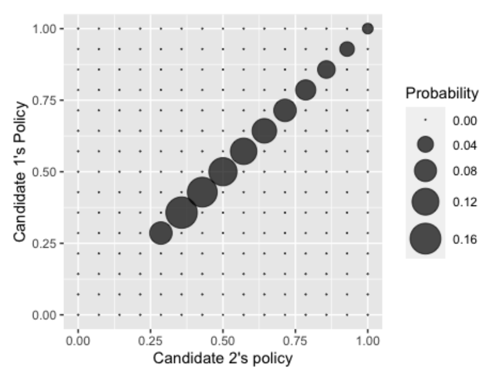
Figure 5 illustrates the joint distribution of candidates' policy platforms given the Sender's optimal strategy, under various priors. Note that points outside the 45-degree line are assigned zero probability, which indicates that candidates always choose identical policy platforms.



(a) Uniform prior



(b) Prior with increasing density



(c) Prior with decreasing density

Figure 5: Joint distribution of candidates' policies under private information design, assuming u_S is linear.

4 Predictions and Welfare

Downs (1957)'s median voter theorem predicts that candidates will converge to the median voter's most preferred policy platform. The present setup differs in two respects: the addition of uncertainty regarding voter preferences and the informed, extremely biased, Sender. Without the biased Sender, any uncertainty that is symmetric across candidates would result in candidates converging to $\theta_{\frac{1}{2}}$, their best guess for θ. The same may not be true, however, in the presence of the Sender.

Under cheap talk, the Sender's incentives do not allow her to publicly impart any information truthfully. In fact, any information transmission must be completely asymmetric, favoring one candidate only. As a result, not only do candidates often diverge on the policy spectrum, but the informed candidate sometimes chooses policies that are biased upwards. One might naturally presume that candidate polarization that results in policy bias must hurt voter welfare. The following example demonstrates this need not be the case.

Suppose that voter utilities take the form of a quadratic-loss function, with bliss points symmetrically distributed around θ. In other words, the state θ shifts the entire distribution of voters.[15] Then, the utilitarian social welfare function is the quadratic loss around θ. Suppose also that θ is uniformly distributed. We can use the results in Section 2.3 to derive the following result.

LEMMA 12. Voter welfare under cheap talk (uniform prior). *If $\theta \sim Uniform[0,1]$, any informative cheap talk equilibrium leads to strictly lower $E(a_w - \theta)^2$ than under the uninformative equilibrium.*

Lemma 12 implies that, under the assumptions above, the presence of the Sender always increases average voter welfare. The quadratic loss function also conveniently

[15] For example, θ is an economic or ideological shock that affects all voters equally.

allows us to interpret this result in terms of the bias and precision of policy around θ. Under cheap talk equilibrium, any information transmission decreases the variance of policy around θ more than it increases its bias.

The intuition behind this result involves a wider choice offered to voters under candidate divergence. Indeed, when candidates diverge, the winning policy depends directly on θ. This ensures that policy cannot stray too far from the median voter's preference. In the Downs (1957) equilibrium, on the other hand, it is candidates' common *belief* about voter preferences that determines policy — if candidates had incorrect beliefs, the election would not reveal their mistake, as voters are not given a choice. This suggests that, when voter preferences are uncertain, there may be welfare gains from having a variety of candidate policies, even at the cost of centrism.

Under public information design, the opposite result prevails. Candidates must always converge to the same policy platform, which is biased upward half of the time. The following result allows us to compare the average voter welfare under public information design to the cheap talk case described above.

LEMMA 13. Voter welfare under public information design. *The optimal Sender strategy described in Theorem 1 (or any optimal pairwise matching of states above and below $\theta_{\frac{1}{2}}$) results in a mean-preserving spread of $|a_w - \theta|$, compared to the equilibrium with no information.*

Corollary. *For any strictly concave social welfare function $W(|a_w - \theta|)$, public information design strictly decreases expected voter welfare compared to no information.*

Under public information design, the expected absolute distance between the winning policy and θ is the same as under no information. Overall, the precision of policy around θ is no better, yet the bias is strictly higher.[16] This is true even though candidates always choose a median of their posterior belief about θ.

Lemma 13 implies that public information design strictly worsens average voter welfare, compared to no information. This is true not only under quadratic loss and uniform prior, but with any prior distribution as long as average voter welfare is concave in $|a_w - \theta|$.

These welfare results have a number of policy implications. For instance, they suggest that disclosure regulations may have a perverse impact on voter welfare. Requiring candidates to disclose their private communications with lobbyists and advisers may remove a potentially beneficial source of information. On the other hand, public information design, which always leads to a larger bias in policy, is unhindered by disclosure regulations. If anything, disclosure obligations are a source of commitment power if the Sender can decide what information to collect.

Recent efforts have been made to publish reliable measures of media bias, as well as information on the activities of lobbying organizations. The model suggests that this may not be effective at curbing their influence on policy. Indeed, the Sender's bias is common knowledge in this model, just as in most models of strategic communication. The discussion at the end of Appendix I suggests that, instead, a diverse offering of media organizations and competing information sources may be more effective at ensuring full information to policymakers.

5 Conclusion

This paper presents a model of informational lobbying in a Downsian election. Contrasting results are obtained depending on the commitment power of the biased

[16]The bias is given by $E(\theta \mid \theta > \theta_{\frac{1}{2}}) - E(\theta)$.

informant.

I show that a Sender with extreme bias and no commitment power may only advise one candidate in equilibrium. This leads to informational asymmetries, candidate divergence and potentially biased policy. However, in some cases, the informational benefit of the Sender may outweigh the bias in policy, thus increasing voter welfare.

On the contrary, a Sender with full commitment power may optimally provide public information to influence both candidates equally. The optimal public information design strategy provides very precise information on the degree of voter extremism, yet leads to policy being biased exactly half of the time. In many cases, this strictly worsens average voter welfare.

The results of this paper refute simplistic arguments that equate bipartisanship with unbiasedness, or polarization with lower voter welfare. An extremely biased informant may be bipartisan; a polarizing adviser may benefit voters. The models presented here attempt to shed some light on how election incentives shape the ways in which candidates' information drives them apart or brings them closer together, and how this affects their ability to serve voters' interests.

The models can be extended in numerous directions, some of which are attempted in Appendix I. The extent of the Sender's bias, her information, and the number of Senders particularly come to mind. Any further examinations are left to future research.

References

Bardhi, A. and Y. Guo (2018). Modes of Persuasion Toward Unanimous Consent. *Theoretical Economics 13*(3), 1111–1150.

Bergemann, D. and S. Morris (2019). Information design: A unified perspective. *Journal of Economic Literature 57*(1), 44–95.

Besley, T. and S. Coate (1997). An economic model of representative democracy. *Quarterly Journal of Economics*.

Callander, S. (2005, oct). Electoral competition in heterogeneous districts. *Journal of Political Economy 113*(5), 1116–1145.

Chakraborty, A. and R. Harbaugh (2007). Comparative cheap talk. *Journal of Economic Theory 132*(1), 70–94.

Chakraborty, A. and R. Harbaugh (2010). Persuasion by Cheap Talk. *The American Economic Review 100*(5), 2361–2382.

Crawford, V. P. and J. Sobel (1982, nov). Strategic Information Transmission. *Econometrica 50*(6), 1431.

Downs, A. (1957, apr). An Economic Theory of Political Action in a Democracy. *Journal of Political Economy 65*(2), 135–150.

Farrell, J. and R. Gibbons (1989). Cheap Talk with Two Audiences. *The American Economic Review 79*, 1214–1223.

Friedman, J. N. and R. T. Holden (2008). Optimal gerrymandering: Sometimes pack, but never crack. *American Economic Review 98*(1), 113–144.

Gentzkow, M. and E. Kamenica (2016, may). A Rothschild-Stiglitz Approach to Bayesian Persuasion. In *American Economic Review*, Volume 106, pp. 597–601. American Economic Association.

Gentzkow, M. and E. Kamenica (2017, apr). Disclosure of endogenous information. *Economic Theory Bulletin 5*(1), 47–56.

Gitmez, A. and P. Molavi (2018). Media Capture: A Bayesian Persuasion Approach.

Grossman, G. M. and E. Helpman (2001). *Special interest politics*. MIT.

Heese, C. and S. Lauermann (2019). The Bayes Correlated Equilibria of Large Elections. *Working Paper* (April).

Herbst, S. (1998). *Reading Public Opinion: How Political Actors View the Democratic Process*. University of Chicago Press.

How Choon, T. (2020). *Essays on Information Revelation and Political Institutions*. Ph. D. thesis, Boston University.

Kamada, Y. and F. Kojima (2014). Voter preferences, polarization, and electoral policies. *American Economic Journal: Microeconomics 6*(4), 203–236.

Kamenica, E. (2019). Bayesian Persuasion and Information Design. *Annual Review of Economics 11*(1).

Kamenica, E. and M. Gentzkow (2011). Bayesian persuasion. *American Economic Review 101*(6), 2590–2615.

Kartik, N. and R. P. McAfee (2007). Signaling character in electoral competition. *American Economic Review*.

Kartik, N. and R. Van Weelden (2019). Informative cheap talk in elections. *Review of Economic Studies 86*(2), 755–784.

Kolotilin, A. and A. Wolitzky (2020). Assortative Information Disclosure.

Lipnowski, E. and D. Ravid (2017). Cheap Talk with Transparent Motives. *Ssrn* (May), 1–56.

Appendix I: Robustness and Extensions

A Moderately Biased Sender

The model in Section 1 illustrates that political competition can incentivize information transmission by an extremely biased Sender. In this section, I allow the Sender's bias vis-à-vis θ to be moderate, as in Crawford and Sobel (1982). I show that in some cases, the equilibria in Section 1 are also equilibria for smaller bias. In particular, when the prior is uniform, the set of equilibria under extreme bias is a subset of equilibria under moderate bias.

LEMMA 14. *Consider the game in Section 1 with the following modification:*

$$U_S(a_1, a_2; \theta) = -E((a_w - \theta - b)^2 \Big| a_1, a_2; \theta)$$

Also, let $\boldsymbol{\sigma}$ be an equilibrium of the original game, such that the set of inducible policies is has at most two elements, i.e. $|A_R| \leq 2$. Then $\boldsymbol{\sigma}$ is also an equilibrium of the modified game with $b > 0$.

Proof: Since candidates' preferences are unchanged, it suffices to check that the Sender would not deviate. Let $A_R = \{a^\ell, a^h\}$, where $a^\ell < a^h$. When $\theta < \frac{a^\ell + a^h}{2}$, the lower policy a^ℓ wins for sure, so the Sender is indifferent and any message will do. When $\theta > \frac{a^\ell + a^h}{2}$, the median voter prefers a^h and so does the Sender, for any $b > 0$, just as in the original model.

Corollary. *If $\theta \sim Uniform[0, 1]$, then any equilibrium of the original game is also an equilibrium of the modified game with moderate bias.*

B Imperfectly Informed Sender

So far, I have assumed that the Sender perfectly observes the state. However, in reality it is doubtful that anyone could perfectly predict the outcome of an election. In this subsection, I assume that the Sender observes an imperfect signal about θ.

If the Sender's information partitions the state space Θ into intervals — if she can rule out some values of θ — it is intuitive that informative equilibria may exist. This is because the information transmitted to the candidate takes on a similar structure. However, this is not true if the Sender's signals induce posteriors with full support over the state space.

Suppose there is a small probability that the Sender's signal is entirely uninformative. For instance, suppose that the Sender observes the true state with probability $1 - \varepsilon$, but with probability ε the Sender receives a meaningless signal that is uniformly drawn from the state space. Then all equilibria are uninformative.

To see why, remember that the Sender's credibility arises from pragmatism: there is no point in making a false recommendation if that policy will be defeated for sure. However, if the Sender's information is imprecise, any policy could win with a small probability! Even if the Sender's information suggests that the lowest policy platform will win, there is always a small chance that an exaggerated recommendation will bear fruit. This creates a small misalignment between the Sender and Receiver: the Sender would like to recommend a higher policy as long as it has any chance at all of winning, whereas the candidate is more cautious and prefers policies that are most likely to win.

However, small modifications to the model are sufficient to resolve this issue. Below, I consider one such modification: a small cost of sending a message. (The cost need not depend on θ). This assumption is entirely plausible in most real world situations. In addition, I assume at least one candidate has a very slight preference for higher policies.

In this model, the existence of informative equilibria depends on the precision of the Sender's information. This differs from Crawford and Sobel (1982), where agents care only about the posterior mean, regardless of the distribution. My model suggests that informants with precise information need not do much to convince politicians, whereas equally biased Senders with imprecise signals have a greater incentive to lie, and therefore need to go to greater lengths to convey similar information.

Messages are *almost* costless

Suppose that, instead of being completely costless, all messages have a small cost $c > 0$ to the Sender. However, there is one message, $m^0 \in M$, that costs nothing — interpret that as "no message". Additionally, assume that the candidate has a small bias towards high policies.

First, if the Sender is perfectly informed, the equilibria that remain are exactly the interval equilibria, where each message denotes an interval of the state space. Furthermore, "no message" must induce the lowest policy.

If instead the Sender receives an almost perfect signal, i.e. $\varepsilon \to 0$, the incentive to exaggerate is outweighed by the cost of the message. The Sender knows that the high policy could win with non-zero probability, but that event is so unlikely that it is not worth sending a message. For the candidate, receiving a message means that a high policy is very likely to win, however there is still a small chance that the Sender is wrong, and voters truly prefer a lower policy. If the candidate only cared about winning, they would deviate and choose a lower policy, just in case the Sender was wrong. However, a small upward bias will make sure they follow the recommendation.

LEMMA 15. *Suppose that $\boldsymbol{\sigma}$ is an interval equilibrium of the original cheap-talk game. Fix small $\beta, c > 0$. Consider the following modifications to the model.*

1. *Add a small cost of sending a message, and small candidate bias.*

$$U_S(\boldsymbol{\sigma};\theta) = E(u_S(a_w) \mid a_1, a_2; \theta) - c \sum_i \mathbb{1}\{m_i \neq m^0\}$$

$$U_R(\boldsymbol{\sigma};\theta) = Pr(w = R \mid \boldsymbol{\sigma};\theta) + \beta\ \sigma_i(\sigma_{SR}(\theta))$$

   Then, $\boldsymbol{\sigma}$ is still an equilibrium if $\sigma_R(m^0) = a_U$.

2. *Let preferences be as stated above, and let the Sender observe the following signal s instead of θ, with the Sender's strategy now being a function of s.*

$$s = \begin{cases} \theta & w.p.\ 1-\varepsilon, \\ x \sim Uniform[0,1] & w.p.\ \varepsilon. \end{cases}$$

   Then, $\boldsymbol{\sigma}$ is still an equilibrium if $\sigma_R(m^0) = a_U$ and $\varepsilon \to 0$.

The proof involves checking that the Sender and candidate R have no incentive to deviate. In Lemma 15.1, a small c and small β have a small enough effect on their utilities to keep deviations unprofitable. (Both players' incentive constraints are not binding in any situation where c and β would worsen them). In Lemma 15.2, the Sender's and R's additional gain from deviating is proportional to ε, whereas the cost is proportional to c and β respectively. If ε is small enough relative to c and β, there is no incentive to deviate.

C Two Senders with Opposite Biases

Suppose that, instead of one extremely biased Sender, there were two Senders with diametrically opposed preferences (increasing and decreasing) in policy. Then, it is trivially true that perfect revelation by both Senders is supported by an equilibrium. Consider the following strategy profile. S_1 and S_2 each perfectly reveal the state. Candidate 1 always chooses a policy equal to the state revealed by S_1 and Candidate 2 likewise listens to S_2. Then neither Sender has an incentive to deviate, as there is always at least one candidate who chooses a policy equal to the median voter's bliss point. Therefore that policy is guaranteed to win. Neither candidate has an incentive to deviate, as deviating guarantees a loss.

This setting illustrates the ideal confrontational system at the basis of most democratic institutions, and arrives at the same prediction as the median voter theorem. The result, however, rests on a number of strict assumptions, chiefly that both Senders have access to the same precise information. The model presented in Chapter One, on the other hand, is more applicable whenever the special interests on one side of the argument have a disproportionate informational advantage over the other. In the real world, this arises naturally due to differences in expertise, wealth, or the ability to mobilize resources.

Appendix II: Proofs

LEMMA 1: Public messages are uninformative. *If we restrict the Sender's strategy to public messages (i.e. $\sigma_{S1} = \sigma_{S2}$ always), then in any equilibrium, both candidates choose $\theta_{\frac{1}{2}}$ regardless of messages.*

The intuition is as follows: for small θ, only the lower policy (between the two candidates) matters, so S always induces the highest such policy. Therefore, the lower policy is unique. For some moderate θ, only the higher policy (between the two candidates) matters, and so S always induces the highest such policy. Then, the higher and lower policy cannot both be posterior medians unless they are equal.

Fix an equilibrium. By the median voter theorem, each candidate must, given each message, choose a policy platform among the set of medians given the posterior, and win with probability half. Formally,

$$\text{For each } i \in \{1,2\} \text{ and for each } m \in M, \quad \sigma_i(m) \in \theta_{\frac{1}{2}}(m)$$

where $\theta_{\frac{1}{2}}(m) \equiv \left\{\hat{\theta} \in \Theta : Pr(\theta \leq \hat{\theta} \,|\, m) \geq \frac{1}{2} \text{ and } Pr(\theta \geq \hat{\theta} \,|\, m) \geq \frac{1}{2}\right\}$.

Now define the set of lower policies

$$A^\ell = \{a \in [0,1] : a = \min_i \sigma_i(m) \text{ for some } m\}$$

Now let $\bar{a}^\ell = \sup A^\ell$. Fix any $a \in A^\ell \backslash \{\bar{a}^\ell\}$. For all $\theta \leq a$, the Sender strictly prefers to induce (something arbitrarily close to) $\bar{a}^\ell$ rather than a, because in both cases the lower policy (between the two candidates) wins for sure. But then $Pr(\theta \leq a|\, a \text{ is induced}) = 0$ and therefore a cannot be the median of the posterior given the message that induces a.

Therefore, the only possibility is that $A^\ell = \{a^\ell\}$ is a singleton. This means that, after any message, at least one of the candidates must choose a^ℓ. We need to show that both candidates always choose a^ℓ.

Suppose not. Define $a^h = \sup\{a \in [0,1] : a = \sigma_i(m) \text{ for some } m, i\}$ to be the highest policy on the equilibrium path. This policy must be a maximum, otherwise when $\theta = 1$, the Sender's optimal message does not exist. For all $\theta \in (\frac{a^\ell + a^h}{2}, a^h)$, the Sender must induce a^h since it will win for sure. But a^ℓ and a^h cannot both be medians of the posterior, since θ lies between them with non-zero probability. Contradiction.

The last step is to show that the unique policy chosen, a, must be equal to $\theta_{\frac{1}{2}}$, the median of F. Suppose not, then given some message the posterior must place unequal mass above and below a. Then each candidate can deviate by choosing instead a median of the posterior, winning with probability strictly higher than half.

LEMMA 2. One candidate does not respond to messages. *On the equilibrium path, at least one candidate's strategy must be independent of messages, i.e.* $\exists\, U \in \{1,2\}$ *such that* $\sigma_U(\sigma_{SU}(\theta)) = a_U \quad \forall \theta$.

Suppose, by contradiction, that the Sender induces at least two distinct policy platforms, $a_i^\ell < a_i^h$, for each of the candidates in some equilibrium. W.l.o.g., let $a_i^h = \sup A_i$, where $A_i = \{a_i : \exists\, \theta \text{ s.t. } a_i = \sigma_i(\sigma_{Si}(\theta))\}$.

First, a_i^h must be induced for some θ and some i (i.e. a_i^h is a max, not just a sup, for some i), otherwise the Sender's best response does not exist when $\theta \geq \max\{a_1^h, a_2^h\}$. Fix that i. For $j \neq i$, the Sender induces a_j^ℓ only if it loses to a_i^h. (If for some θ, a_j^ℓ has non-zero chance of winning, it is strictly better for the Sender to induce a_i^h and something arbitrarily close to a_j^h.) Then after receiving message a_j^ℓ, candidate j knows that i is playing a_i^h and that a_j^ℓ must lose for sure. So j can deviate profitably to playing a_i^h and winning with probability $\frac{1}{2}$.

LEMMA 3. Uninformed candidate chooses lowest on-the-equilibrium-path policy. *Let U denote the candidate whose strategy is independent of messages, and let R denote the other. Then* $a_U = \inf A_R$, *where* $A_R = \{a_R : \exists\, \theta \text{ s.t. } a_R = \sigma_R(\sigma_{SR}(\theta))\}$ *is the set of all policies that R may choose after receiving some message.*

Denote $a_R^\ell = \inf A_R$.

Suppose $a_U < a_R^\ell$. Then for any $\theta > \frac{a_U + a_R^\ell}{2}$, R must win, otherwise the Sender is not playing best response. Then U can deviate profitably by choosing $a'_U = a_R^\ell - \varepsilon$, where ε is arbitrarily small.

Suppose $a_U > a_R^\ell$. Then there exist some θ for which the Sender induces $a_R < a_U$. Fix such an a_R. If given a_R is induced, R never wins, then R can do strictly better by choosing a_U. If R sometimes wins, then the Sender can deviate by inducing something strictly higher than a_R when a_R would have won.

LEMMA 4. Lowest policy is no higher than the median of F. $a_U \leq \theta_{\frac{1}{2}}$.

Suppose $a_U > \theta_{\frac{1}{2}}$. First note that a_U must be a median of the posterior conditional on the Sender recommending a_U to R, otherwise R could deviate by a small amount and win with probability strictly greater than 0.5. Now there must be some $a_R \in A_R \backslash \{a_U\}$ such that the posterior (after receiving the message that induces a_R) assigns greater than half probability to $\theta < a_U$. Since $a_R > a_U$ by Lemma 3, R wins less than half of the time, and R can deviate to some policy below a_U and win strictly more than half the time.

LEMMA 5. Finitely many messages. *Assume* $f(\theta) > 0$ *for all* $\theta \in (0, 1)$. *Then,* A_R *is finite.*

Note: $f(\theta) > 0$ for $\theta \in (0, 1)$ is stronger than full support, but is satisfied under full support and single-peakedness. Note that $f(\theta)$ is allowed to be 0 at the extremes.

First we make the following claim.

Claim: For any real number $a > a_U$,

$$\sup\{a_R \in A_R : a_R < a\} < a \quad \text{and is a } \max .$$

i.e. there exists a highest element below a.

First, suppose $\sup\{a_R \in A_R : a_R < a\} = a$ for some a. Then when $\theta = \frac{a_U+a}{2}$, S's optimal strategy does not exist, because S can deviate to inducing a policy that is arbitrarily close below a that can win for sure. Now, suppose that the sup is not a max. Then set $a' = \sup\{a_R \in A_R : a_R < a\}$ and apply the first part of the claim. Contradiction.

Suppose A_R is infinite. Construct a series of arbitrarily small upward deviations for U, the uninformed candidate, as follows.

Define the function $h : (a_U, 1] \to A_R \backslash \{a_U\}$ by saying $h(a) = \max\{a_R \in A_R : a_R < a\}$. Starting with $a_0 = h(1)$, let $a_t = h(a_{t-1})$ for all t. Since A_R is infinite, this gives us an infinite series of strictly decreasing elements, from the highest element of A_R to the lowest. Now let $\epsilon_t = \frac{1}{2} \min_{1 \leq k \leq t}\{a_{t-1} - a_t\} > 0$.

By deviating upwards to $a_U + \epsilon_t$, U loses approximately $\epsilon_t \frac{1}{2} f(a_U)$, which occurs when θ happens to be just below $\frac{a_U+a_U+\epsilon}{2}$. However, U gains at least $\epsilon_t \sum_{k=1}^{t} f(\frac{a_k+a_U}{2})$. (For θ just above $\frac{a_k+a_U}{2}$, R would usually just win, but now U wins instead). As $\epsilon_t \to 0$, this sum grows to infinity, and the deviation is profitable.

LEMMA 6. Two messages. *If $\theta \sim Uniform[0, 1]$, then A_R (the set of possible policy platforms on the equilibrium path) is at most a doubleton.*

Suppose not, i.e. $|A_R| \geq 3$. Then U can deviate by choosing $a'_U = a_U + \varepsilon$ instead. From Lemmas 3 and 5, we know that $a_U = \min A_R$. Also, a_U is the median of the posterior on θ conditional on R choosing a_U, otherwise R would deviate. As $\epsilon \to 0$, U's gain from deviating converges to:

$$\varepsilon\Big(-\frac{1}{2}f(a_U) + \frac{1}{2}\sum_{a_R \in A_R \setminus \{a_U\}} f(\frac{a_U + a_R}{2})\Big) \quad = \quad \varepsilon\Big(-\frac{1}{2} + \frac{1}{2}(|A_R| - 1)\Big) > 0$$

The first term in brackets refers to the loss from deviating when R chooses a_U and $\theta < \frac{a_U + a'_U}{2}$. The second term refers to the gain when $\sigma_R(\sigma_{SR}(\theta)) \neq a_U$. The Sender recommends $a_R \in A_R \setminus \{a_U\}$ if $\theta > \frac{a_U + a_R}{2}$, i.e. whenever a_R wins against a_U. So for U, deviating by ε means pushing the cutoff for a win by $\frac{1}{2}\varepsilon$ and capturing those θ between the old and new cutoffs. There is a cutoff for each $a_R \in A_R \setminus \{a_U\}$, so add up the gains from moving all of them.

LEMMA 7. Interval equilibria. *If $\theta \sim Uniform[0,1]$, the following is an equilibrium iff $a^\ell \in [\frac{1}{4}, \frac{1}{3}]$ and $a^h = 3a^\ell$.*

Fix some arbitrary $m_U \in M$ and a_ℓ, a_h satisfying the conditions given above. Also fix a partition $\{M^\ell, M^h\}$ of M, and arbitrary elements $m^\ell \in M^\ell$ and $m^h \in M^h$.

$$\sigma_S(\theta) = \begin{cases} (m_U, m^\ell) & \textit{if } \theta < \frac{a^\ell + a^h}{2}, \\ (m_U, m^h) & \textit{otherwise}. \end{cases}$$

$$\sigma_U(m) = a_U = a^\ell \quad \textit{for all } m \in M, \quad \textit{with prior belief after every } m.$$

$$\sigma_R(m) = \begin{cases} a^\ell & \textit{for all } m \in M^\ell, \quad \textit{with belief } \theta \sim Uniform[0, \frac{a^\ell + a^h}{2}], \\ a^h & \textit{for all } m \in M^h, \quad \textit{with belief } \theta \sim Uniform[\frac{a^\ell + a^h}{2}, 1]. \end{cases}$$

It is easy to show that each player is playing best response given others' strategies. When $\theta < \frac{a^\ell + a^h}{2}$, a^ℓ wins for sure, therefore the Sender is indifferent between all messages. When $\theta \geq \frac{a^\ell + a^h}{2}$, a^h can win so it is strictly better to induce a^h.

a^ℓ is optimal for U because any other policy gives a strictly (weakly if $a^\ell = \frac{1}{4}$) lower probability of winning. For R, there are two possible posterior beliefs about θ. After receiving message m^ℓ, the posterior is a uniform distribution over $[0, \frac{a^\ell + a^h}{2})$, and a_U is the median of that posterior, so R's best response is a_U. After receiving message m^h, R knows $\theta > \frac{a^\ell + a^h}{2}$, so a^h guarantees a win with probability 1.

LEMMA 8. Sender-optimal equilibrium. *If $\theta \sim Uniform[0, 1]$, then the following is an equilibrium, and it achieves the highest equilibrium expected payoff for the Sender.*

Fix some arbitrary $m_U \in M$. Also fix a partition $\{M^1, M^2\}$ of M, and arbitrary elements $m^1 \in M^1$ and $m^2 \in M^2$.

$$\sigma_S(\theta) = \begin{cases} (m_U, m^1) & \textit{if } \theta \in [0.25, 0.75), \\ (m_U, m^2) & \textit{otherwise}. \end{cases}$$

$$\sigma_U(m) = 0.5 \quad \textit{for all } m \in M, \quad \textit{with prior belief after every } m.$$

$$\sigma_R(m) = \begin{cases} 0.5 & \textit{for all } m \in M^1, \quad \textit{with belief } \theta \sim Uniform[0.25, 0.75), \\ 1 & \textit{for all } m \in M^2, \quad \textit{with posterior given } \theta \in [0, 0.25) \cup [0.75, 1]. \end{cases}$$

It is easy to show that the above is an equilibrium. To show that it is the best equilibrium for the Sender, we refer to Lemma 6 which implies that that A_R is a doubleton. Let $A_R = \{a^\ell, a^h\}$. First note that, comparing across equilibria, the Sender's utility is increasing in both a^ℓ and a^h. In the above equilibrium, $a^h = 1$, the highest possible number. $a^\ell = \frac{1}{2}$ is also the highest possible value, by Lemma 4.

LEMMA 9. Sender and informed candidate both benefit from cheap talk. *If $\theta \sim Uniform[0,1]$ and u_S is linear, then any informative equilibrium gives weakly higher utility to the Sender and to candidate R than the uninformative equilibrium.*

In the uninformative equilibrium, the implemented policy is $\frac{1}{2}$. This gives utilities $u_S = \frac{1}{2}$ and $u_R = \frac{1}{2}$.

In any informative equilibrium, the Sender's utility is no lower than $\frac{1}{2}$ because $\frac{a^h+a^\ell}{2} \geq \frac{1}{2}$. (Suppose $\frac{a^h+a^\ell}{2} < \frac{1}{2}$, then U could deviate to slightly above a^h, which increases U's winning probability from a^ℓ to $1-a^h$). R's utility is at least $\frac{1}{2}$ because he can deviate to a constant policy of $\frac{1}{2}$, which guarantees him a utility of $\frac{1}{2}$.

LEMMA 10. Increasing, decreasing and symmetric prior.

(a) If f is strictly increasing in θ, all equilibria are uninformative.

(b) If f is weakly decreasing in θ, an informative equilibrium exists.

(c) If f is single-peaked and symmetric around $\theta = \frac{1}{2}$, an informative equilibrium exists. [17]

To prove (a), suppose there was a non-babbling equilibrium. In that equilibrium, for all $a_R \in A_R \backslash \{a_U\}$,

$$f\left(\frac{a_U + a_R}{2}\right) > f(a_U)$$

[17] Note that the uniform distribution fits both (b) and (c). In fact, the uniform distribution is at the boundary between increasing and decreasing, so that *all* informative equilibria are doubleton partitions — the coarsest possible information.

Now consider a small deviation for U, where U chooses $a_U + \varepsilon$ instead of a_U. As $\varepsilon \to 0$, the gain from deviating converges to

$$\varepsilon\Big(-\frac{1}{2} f(a_U) + \frac{1}{2} \sum_{a_R \in A_R \setminus \{a_U\}} f(\frac{a_U + a_R}{2})\Big) > 0$$

Contradiction.

Existence is proved by constructing an informative equilibrium with two policy platforms on the equilibrium path. In (b) where f is decreasing, this takes the form of an interval equilibrium where the Sender reveals whether θ is above or below $\frac{1}{2}$. Candidate R's policies are equidistant from $\frac{1}{2}$, with the lower policy being $a^\ell = \text{median}\,(\theta \mid \theta < \frac{1}{2})$. Under symmetric single-peaked f in (c), an equilibrium similar to Lemma 8 exists where the Sender reveals whether θ is in the interval $[\frac{1}{4}, \frac{3}{4})$ or outside it. On the equilibrium path, the policies are $\frac{1}{2}$ and 1 respectively.

THEOREM 1. Optimal public information design. *The following signal structure is a solution to the Sender's public information design problem, and achieves the maximal utility equal to $E(u_S(\theta) \mid \theta > \theta_{\frac{1}{2}})$.*

$$m^*(\theta) = \Big| F^{-1}(\theta) - \frac{1}{2} \Big|$$

$$p(m|\theta) = \begin{cases} 1 & \textit{if} \quad m = m^*(\theta), \\ 0 & \textit{otherwise} \end{cases}$$

$$\sigma_i(m) = F^{-1}(\frac{1}{2} + m) \quad \textit{for all } i \in \{1, 2\},\ m \in M$$

First, we can show that this signal structure achieves utility equal to $E(u_S(\theta) \mid \theta >$

$\theta_{\frac{1}{2}}$). First note that, for any random variable x with distribution H, we can define $H(x)$ as another random variable and $H(x) \sim Uniform[0,1]$. Now define $u = u_S(\theta)$ as a random variable with distribution $H(u)$.

Then, we can write down the Sender's expected utility under the above strategy:

$$E(u_S(a_w)) = \int_0^{0.5} H^{-1}(1-t)dt + \int_{0.5}^1 H^{-1}(t)dt$$

By a change of variables, we can rewrite the first term to be equal to the second term:

$$\begin{aligned} E(u_S(a_w)) &= 2\int_{0.5}^1 H^{-1}(t)dt \\ &= 2 \times \int_{u_S(\theta_{\frac{1}{2}})}^{u_S(1)} u dH(u) \\ &= E(u_S(\theta) \mid \theta > \theta_{\frac{1}{2}}) \end{aligned}$$

Then we need to show that $E(u_S(\theta) \mid \theta > \theta_{\frac{1}{2}})$ is, in fact, an upper bound on the Sender's utility. Do this in two steps:

1. Define a subset of F to be a function $G : \Theta \longrightarrow [0,1]$ s.t. for all $x \geq x'$,

   $$G(x) - G(x') \leq F(x) - F(x').$$

   Then state that

   $$\text{If } G(1) = \frac{1}{2}, \text{ then } \frac{\int_0^1 u_S(\theta)dG(\theta)}{\frac{1}{2}} \leq E(u_S(\theta) \mid \theta > \theta_{\frac{1}{2}})$$

   In other words, take any half of the θ population, and their average is no higher than the average of the top half of the population.

Proof: Define the top half of F to be the subset $F_{\frac{1}{2}}$ such that $F_{\frac{1}{2}}(\theta) \equiv \min\{0, F(\theta) - \frac{1}{2}\}$.

($F_{\frac{1}{2}}$ assigns zero probability to all θ below the median of F, but preserves density f for all θ above the median). It is easy to show that, if $G \neq F_{\frac{1}{2}}$ is a subset of F and if $G(1) = \frac{1}{2}$, then $F_{\frac{1}{2}}$ FOSD G. That's because $F_{\frac{1}{2}}$ already assigns the maximum possible probability to high values of θ.

2. Now suppose there exists a signal structure that achieves a utility strictly higher than $E(u_S(\theta) \mid \theta > \theta_{\frac{1}{2}})$. Then construct a subset of F which consists of all the realizations of θ above each posterior median. That subset has probability $\frac{1}{2}$ and average higher than u, which contradicts 1.

   Specifically, for each posterior belief F^m corresponding to message (signal realization) m, define

$$F^m_{\frac{1}{2}}(\theta) \equiv \min\{0, F^m(\theta) - \frac{1}{2}\}.$$

   Now the Sender's utility is the expectation of the posterior median:

$$u = \int_m u_S(\theta_{\frac{1}{2}}(m))\Pr(m)$$

Since by definition, $F^m_{\frac{1}{2}}(\theta) = 0$ for all $\theta < \theta_{\frac{1}{2}}(m)$, the above expression is smaller than if we replace $u_S(\theta_{\frac{1}{2}}(m))$ with $\frac{\int_0^1 u_S(\theta)\, dF^m_{\frac{1}{2}}(\theta)}{\frac{1}{2}}$, which is the posterior expectation above $\theta_{\frac{1}{2}}(m)$.

Then we can construct a subset of F by combining all the posterior subsets $F^m_{\frac{1}{2}}$. Define $G(\theta) = \int_m F^m_{\frac{1}{2}}(\theta)\Pr(m)\, dm$. We can show trivially that $G(1) = \frac{1}{2}$. We can also show that G is a subset of F, by using the fact that $F(\theta) = \int_m F^m(\theta)\Pr(m)\,dm$.

$$\frac{\int_0^1 u_S(\theta)\,dG(\theta)}{\frac{1}{2}} = \int_m \frac{\int_0^1 u_S(\theta)\ dF^m_{\frac{1}{2}}(\theta)}{\frac{1}{2}} \Pr(m)\,dm$$

We already established that the RHS was larger than u, which is larger than $E(u_S(\theta)\,|\,\theta > \theta_{\frac{1}{2}})$. Contradiction.

LEMMA 11. Commitment versus private messages. *Suppose that $u_S(.)$ is weakly concave and that f is single-peaked and symmetric. Then the Sender's utility under public information design is strictly higher than under any cheap talk equilibrium.*

Under the strategy described in Theorem 1, the Sender's utility conditional on each message m is exactly $u_S(F^{-1}(\frac{1}{2}+m))$. Consider the Sender's utility in a cheap talk equilibrium, conditioning on the same event $\theta = F^{-1}(\frac{1}{2} \pm m)$.

When θ takes on the lower value, $a_w = a_U$. (This is because $a_w = a_U$ for all $\theta \leq \frac{1}{2}$, otherwise U would deviate upwards due to the shape of f.) When θ takes on the higher value, $\theta \geq \frac{a_U+a_w}{2}$, which is a necessary condition for a_w to win. This can be rewritten as $a_w \leq 2\theta - a_U = 2F^{-1}(\frac{1}{2}+m) - a_U$. (Note that the inequality is strict for most m because full revelation is not possible).

Therefore, a_w is on average no more than $F^{-1}(\frac{1}{2}+m)$. Since u_S is concave, this gives the Sender an expected utility lower than $u_S(F^{-1}(\frac{1}{2}+m))$.

LEMMA 12. Voter welfare under cheap talk (uniform prior). *If $\theta \sim Uniform[0,1]$, any informative cheap talk equilibrium leads to strictly lower $E(a_w-\theta)^2$ than under the uninformative equilibrium.*

$E(a_w-\theta)^2 = Var(\theta) = \frac{1}{12}$ under the uninformative equilibrium.

Under any informative equilibrium, let $A_R = \{a^\ell, a^h\}$, using Lemma 6. From

Lemma 4, $a^\ell \leq \frac{1}{2}$. Furthermore, $a^h > \frac{1}{2}$, otherwise U would deviate to $a^h + \varepsilon$. Finally, $a_w = a_\ell$ iff $\theta < \frac{a^h + a^\ell}{2}$, and $a_w = a_h$ otherwise. Under these conditions, simple algebra will yield the result that $E(a_w - \theta)^2 < \frac{1}{12}$.

LEMMA 13. Voter welfare under public information design. *The optimal Sender strategy described in Theorem 1 (or any optimal pairwise matching of states above and below $\theta_{\frac{1}{2}}$) results in a mean-preserving spread of $|a_w - \theta|$, compared to the equilibrium with no information.*

Consider the distribution of $|a_w - \theta|$ under an optimal pairwise matching. The support of each posterior is $\{\theta_\ell, \theta_h\}$, where both values are equally likely and $\theta_\ell \leq \theta_{\frac{1}{2}} \leq \theta_h$. Furthermore, $a_w = \theta_h$. Therefore, $|a_w - \theta|$ takes values 0 and $\theta_h - \theta_\ell$ with equal probability, compared to $\theta_{\frac{1}{2}} - \theta_\ell$ and $\theta_h - \theta_{\frac{1}{2}}$ under no information. Note that the mean of $|a_w - \theta|$ is equal to $(\frac{\theta_h - \theta_\ell}{2})$ in both cases, but the former involves more extreme values.